

UNIT 4

NETWORK ANALYSIS

OUTLINE

- Session 4.1: Introduction
- Session 4.2: Critical path analysis CPM limitations
- Session 4.3: Terminologies
- Session 4.4: Notations for drawing network diagrams
- Session 4.5: Constructing network diagrams
- Session 4.6: Activity on node (AON) diagrams
- Session 4.7: Critical path
- Session 4.8: Early times
- Session 4.9: Late times
- Session 4.10: Float
- Session 4.11: Project time reduction
- Session 4.12: Activity on arc (AOA) diagrams
- Session 4.13: Program Evaluation and Review technique(PERT)

OBJECTIVES

By the end of this unit you should be able to:

1. Distinguish between AOA and AON notations
2. Draw network diagrams using AON and AOA notations.
3. Calculate normal project completion time and cost
4. Calculate and explain floats, early and late times of activities
5. Explain crashing and calculate crash cost of an activity
6. Distinguish between CPM and PERT

Note: In order to achieve these objectives, you need to spend a minimum of four (4) hours and a maximum of six (6) hours working through the sessions.

SESSION 4.1: INTRODUCTION

This is a tool for organizing and planning large-scale projects, which consists of a list of smaller tasks that need to be completed at various stages in order for the project to progress and to be completed on time. Network analysis is a generic term for a family of related techniques developed to aid management to plan and control projects. These techniques show the interrelationship of the various jobs or tasks, which make up the overall project and clearly identify the critical parts of the project. They can provide planning and control information on the time, cost and resource aspects of a project.

Network analysis is likely to be of most value where projects are:

1. Complex. That is, they contain many related and interdependent activities.
2. Large, that is many types of facilities (high capital investment, many personnel, etc) are involved.
3. Where restrictions exist. That is where projects have to be completed within stipulated time or cost limits, or where some or all of the resources are limited.

Such projects might be for example:

1. The construction of a transport link;
2. Preparing a dinner party (organizing a wedding)
3. Launching a new product. Etc.

Each of these projects involves a large number of activities, which all need to be completed in a certain order before the whole project is finished. Some activities can be performed simultaneously, whilst others can only be tackled once other activities have finished. For example if you are planning a building, clearly the plastering of the internal walls can be undertaken at the same time as the grounds or landscaped. However, the walls cannot be built until the foundations are laid.

Here we can see a natural precedence relation between the activities and as each activity can only be completed in a certain time scale, the project manager faces the dilemma of carrying out all the activities in the best time and at reasonable cost so that the project completes on time and meets the budget constraints.

Businesses are constantly working on projects and from your own experience, you will know that management of these projects is very important. There are a number of methods that can be used to successfully manage a project, but we will be focusing on one particular aspect - time: extremely important to all businesses we acknowledge.

An unexpected delay or similar time crisis can put a business under a lot of pressure. So, how can you plan your time and identify where delays could occur, leaving you to make decisions to compensate for such situations? A useful tool to use would be the Critical Path Analysis(CPA)

SESSION 4.2: CRITICAL PATH ANALYSIS

Critical Path Analysis can be defined as the logical sequencing of a series of events necessary for a successful research project in such a manner that the most efficient route to some culmination point can be calculated. Consequently, the critical path technique has a multitude of uses: (a) As an aid in time management, (b) as a provider of ongoing data for assessing progress, and (c) to give the researcher or program planner considerable information for decision-making.

The critical path analysis is a tool that illustrates the individual tasks of a project highlighting the expected starting and finishing tasks of each. More precisely, the critical path analysis can be used to:

- Estimate the minimum/maximum time that tasks will be started and completed
- Estimate the minimum time that the whole project will take to complete
- Identify if resources are not being used effectively
- Make aware any tasks that could create a possible delay
- Sequence logically of activities that must be made

Ultimately, the critical path analysis will suggest which tasks are critical to keep on time anticipating that the delay in any one of the tasks will delay the whole project.

The critical path analysis is hard to explain in more detail without the use of diagrams, and so the working example in the following section will make it all clearer.

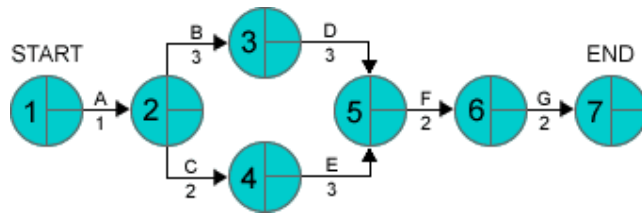
Illustration

Jubilant Limited have decided to carry out some research to ultimately create a selling strategy for their new product. They have decided to create a questionnaire, which they will issue to the public personally. In addition, they will use a mail shot to send out a similar survey to get the opinions of those that live outside of the area.

Each task (A - F) has been given an expected completion time (in weeks). Time is crucial for Jubilant Limited and so the Manager has requested a Critical Path Analysis of the project

Task	Description	Order/Logic	Time
A	Plan Primary Research	To be completed first	1 wks
B	Prepare Mail Shot (Postal Survey)	Start when A is complete	3 wks
C	Prepare Questionnaire	Start when A is complete	2 wks
D	Send and Wait for Mail Shot Replies	Start when B is complete	3 wks
E	Issue Questionnaire	Start when C is complete	3 wks
F	Compile and Analyze Results	Start when D & E is complete	2 wks
G	Plan Selling Campaign	Start when D, E & F is complete	2 wks

Now, take a good look at the diagram below. Each circle (Node) will be used to enter specific data. The numbers currently in the nodes (1 - 7) are only there to make following the diagram easier - nothing more. The arrows represent the tasks and each is given their respective completion times.



Before we move on, look at the above table again and ensure that you understand what is being said by linking it back to the diagram. Basically, all tasks cannot start until the previous task has been completed. This is not true for tasks B and C which can start at the same time - this is the only tricky area.

Each of these projects involves a large number of activities, which all need to be completed in a certain order before the whole project is finished. Some activities can be performed simultaneously, whilst others can only be tackled once other activities have finished. For example if you are planning a building, clearly the plastering of the internal walls can be undertaken at the same time as the grounds or landscaped. However, the walls cannot be built until the foundations are laid. Here we can see a natural precedence relation between the activities, and as each activity can only be completed in a certain time scale, the project manager faces the dilemma of carrying out all the activities in the best time and at reasonable cost so that the project completes on time and meets the budget constraints.

For each activity, there is a set of activities called predecessors of the activity that must be completed before the activity begins. We need a list of all the activities that make up the project. The project is considered complete when all the activities have been completed


4.1.1: CPM Limitations

CPM was developed for complex but fairly routine projects with minimal uncertainty in the project completion times. For less routine projects there is more uncertainty in the completion times, and this uncertainty limits the usefulness of the deterministic CPM model. An alternative to **CPM is the PERT** project planning model, which allows a range of durations to be


specified for each activity. CPM is a deterministic method that uses a fixed time estimate for each activity. While CPM is easy to understand and use, it does not consider the time variations that can have a great impact on the completion time of a complex project.

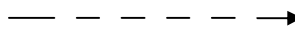
SESSION 4.3: TERMINOLOGIES

1. **Activity:** This is a task or job of work, which takes or consumes time and resources. For example, “Build a wall”, Dig foundations for a building”, “Verify the names of debtors in a sales ledger’, etc.

An activity is represented in a network by an arrow as shown  .

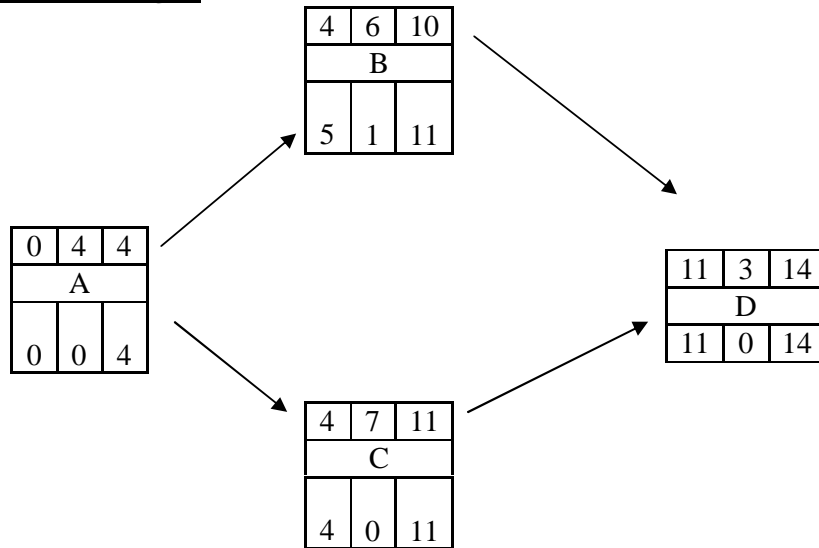
The tail of the arrow indicates where the task begins, and the head where the task ends. The arrow points from left to right and it is not drawn to scale.

2. **Event:** This is a point in time and indicates the “start” or “finish” of an activity or activities. An event is represented in a network by a circle or node as shown .

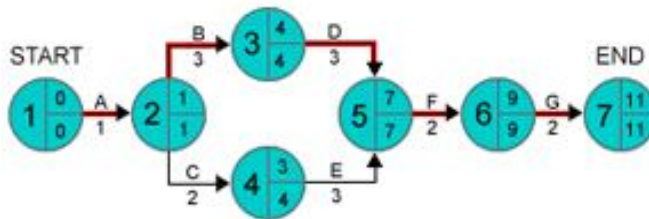
3. **Dummy activity:** This is an activity, which does not consume time or resources. It is used to merely show clear, logical dependencies between activities so as not to violate the rules for drawing networks. It is represented in a network by dotted arrow thus. 

4. **Network:** This is the combination of activities, dummy activities and events in logical sequence according to the rules for drawing networks. Thus a small network might appear as shown below.

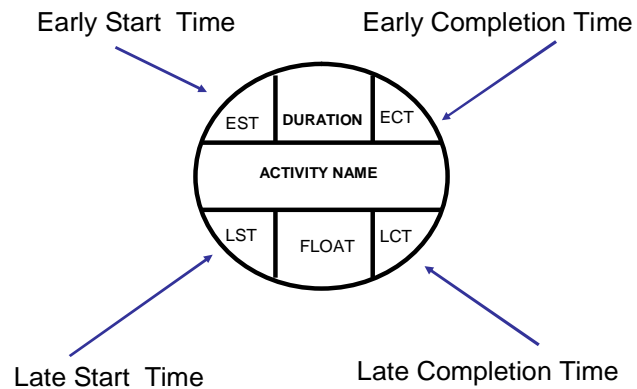
Use of rectangles



Use of circles



Note: The circles/rectangles are divided into sectors as



shown.

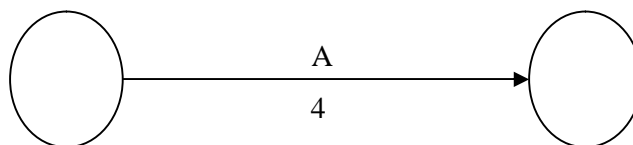
Early Start Time (EST)	Duration	Early Completion Time (ECT)
Activity Description		
Latest Start Time (LST)	Float	Latest Completion Time (LCT)

Note: In a project, an activity is a task that must be performed and an event is a milestone marking the completion of one or more activities. Before an activity can begin, all of its predecessor activities must be completed. Project network models represent activities and milestones by arcs and nodes.

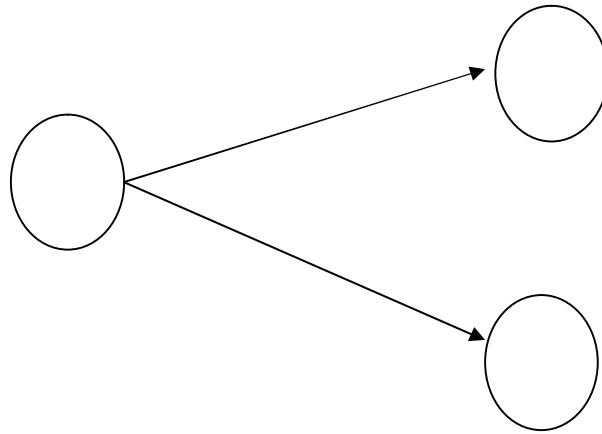
SESSION 4.4: NOTATIONS FOR DRAWING NETWORK DIAGRAMS

A network diagram is a graphical representation of the entire project. Each activity in the project is represented by a circle (node) or rectangle. Arrows are used to indicate sequencing requirements.

There are two main notations used in network diagrams. The first is arrow diagram notation where each activity is represented by an arrow (or a line) joining two circles (or nodes). The nodes represent transitions between activities, which are referred to as events. The duration of the activity is written against the arrow representing the activity. This type of representation is called Activity on Arc (AOA). The diagram below shows activity A in an arrow diagram. The duration of A being 4 days. The nodes are numbered beginning from the “start” node and then move from left to right.



The second notation is the Activity on Node (AON) diagram notation. Here each activity is represented by a circle (or node) and the arcs(lines) represent the events which are the transitions between activities. The duration of the activity is written against the arrow emerging from its node (circle). The diagram below shows activity A in AON notation, which has two successors, B and C. The name of the activity is written in the node(circle) in this case.



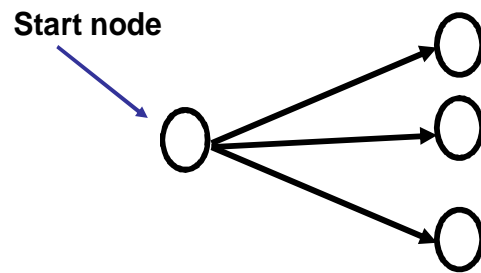
The following rules are used when drawing network diagrams:

- i. Node 1 represents the ‘start’ of the project. An arc or arcs should lead from node 1 to represent such activity that has no predecessor. A network has only one “start” node.
- ii. A node (called the “finish” node) representing the completion of the project should be included in the network. A network has only one “finish” node.
- iii. Number the nodes in the network so that the node representing the completion of an activity always has a larger number than the node representing the beginning of an activity. (There may be more than one numbering scheme)
- iv. An activity should not be represented by more than one arc in the network
- v. Two nodes can be connected by at most one arc. That is activities should not share the same “start” node (tail event) and “finish” node (head event).

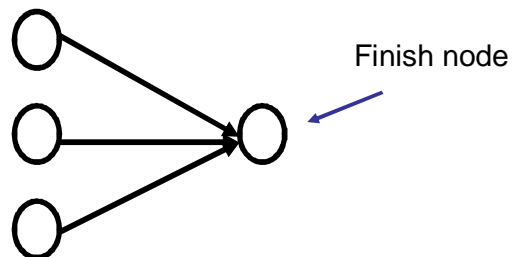
- vi. Every activity must have one preceding event (tail event) and one succeeding event(head event).

Note that

1. Many activities may use the same “start” node (tail event).



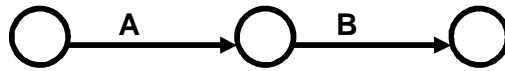
2. Many activities may use the same “finish” node (head event)



Illustrations - 1

(1). Suppose activity A is a predecessor of activity B, then we have:

AOA



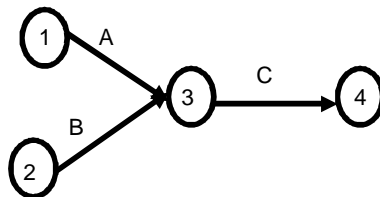
AON



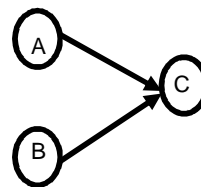
Illustrations - 2

2. Suppose activity A and B must be completed before activity C can begin and that A and B do not start at the same time but are performed simultaneously

AOA



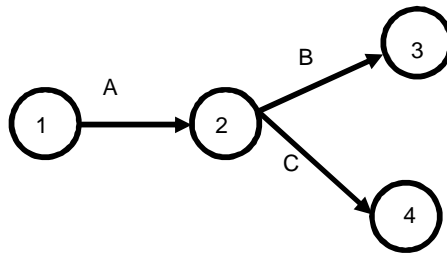
AON



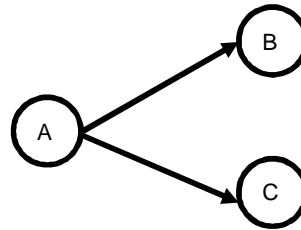
Illustrations - 3

3. Suppose activity A is a predecessor of activities B and C.
(B and C start from the same 'start' node).

AOA



AON



SESSION 4.5: CONSTRUCTING NETWORK DIAGRAMS

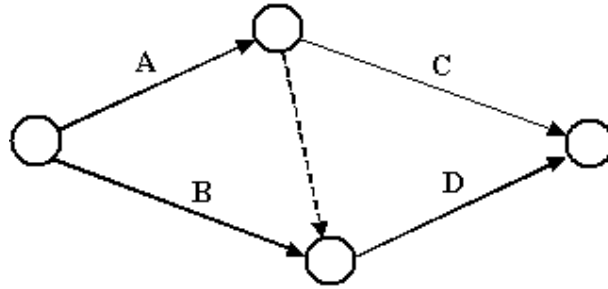
The first stage in constructing a network diagram for a project is to consult with the technical experts and summarise the information that they provide in a table, as in the following simple example.

<u>Activity</u>	<u>Predecessors</u>	<u>Duration</u>
A	None	4 months
B	None	3 months
C	A	2 months
D	A, B	4 months

The **predecessors** impose constraints on the project: C cannot begin until A has been completed, whereas D cannot begin until both A and B have been completed.

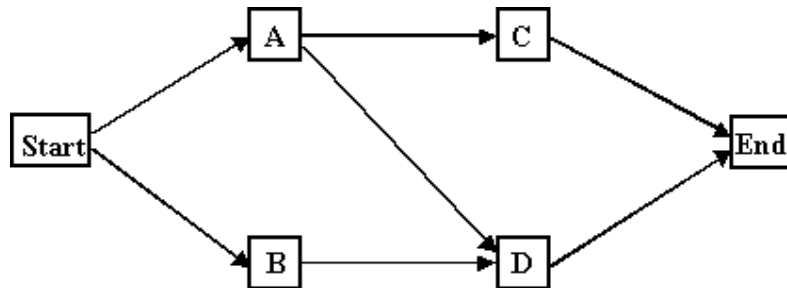
There are two types of network that we can draw for this project:

1. **Activity-on-arrow (AOA)**



The **Dummy Activity** (shown by the dotted line) is needed in the AOA network to show the predecessors correctly, i.e. that D depends on both A and B, but C depends only on A. AOA networks often need several dummy activities. AON networks do not need dummy activities.

2. **Activity-on-node (AON)**



Which type of network is better? Students often find AON networks a little easier to draw, and this can seem like a big advantage when doing it for the first time. However, some of the more advanced types of network analysis are best done using an AOA network. Every well educated student should be able to draw and analyse either, so that they are never thrown into confusion if they come across the one they haven't learnt to use. For people in employment, if your boss has a preference, then that's the one to use.

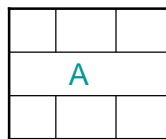
SESSION 4.6: ACTIVITY ON NODE (AON) DIADRAMS

Data below will be used to illustrate the drawing of AON diagram

Example 1: Draw a network diagram using the following data.

Activity	Preceding activity	Duration
A	-	4
B	A	6
C	A	7
D	B, C	3

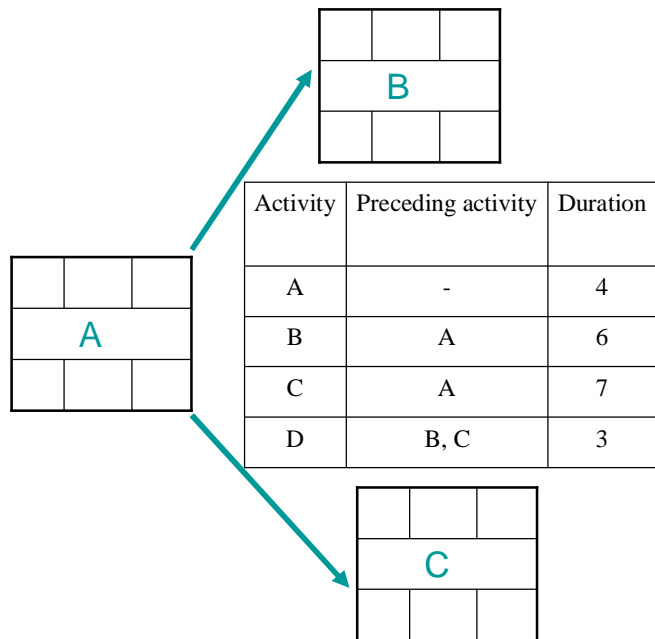
Step 1: Since activity A does not have any predecessor, it uses the first node as shown



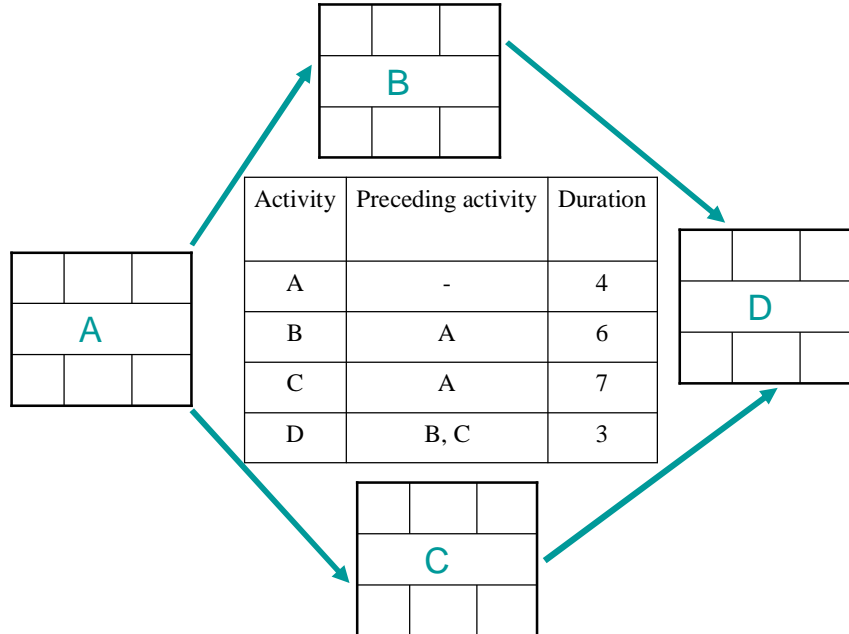
Activity	Preceding activity	Duration
A	-	4
B	A	6
C	A	7
D	B, C	3

Note 1: Node 1 represents the ‘start’ of the project. An arc or arcs should lead from node 1 to represent such activity that has no predecessor. A network has only one “start” node.

Step 2: From activity A, move to activities B, and C



Step 3: From activities B and C, move to activity D



SESSION 4.7: CRITICAL PATH

The critical path of a network gives the shortest time in which the whole project can be completed. It is the chain of activities with the longest duration times.

Certain activities are critical to the on-line completion of the project. For example, if you are planning a dinner party and spend all your time setting the table and forget to put the main dish in the oven, the dinner will be late. This also means that other activities have float or slack time (time to spare) available such as laying the table

From the network diagram, the paths are:

1. A through C to D ($4 + 6 + 3 = 13$)
2. A through B to D ($4 + 7 + 3 = 14$)

The longer of this is A through C to D. Therefore, the critical path is: A through C to D

SESSION 4.8: EARLY TIMES

The Early Start Time (EST) is the earliest time at which the event corresponding to node i can occur. Calculations begin from the 'start' node and move to the 'finish' node. At each node a number is computed representing the Earliest Start Time (EST).

Note:

1. Let EST_i be the earliest start time of event i . If $i=1$ is the start event time of event i , then $EST_i = 0$. That is, the EST of the first activity in any network diagram is zero (0)

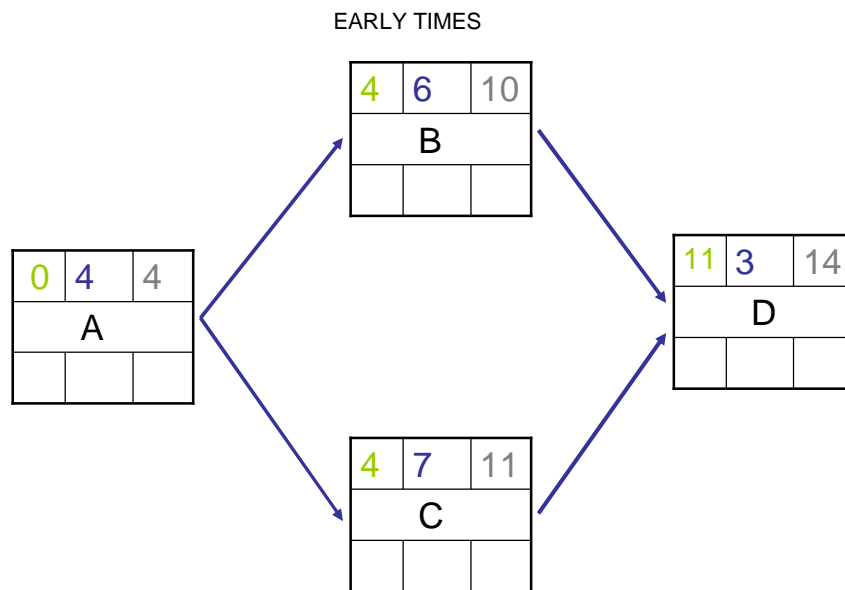
2. Let D_i be the duration of activity i . Then the Early Times calculations are obtained from the formula: $EST_j = \text{Max}(EST_i)$

That is the EST of any activity is equal to the maximum of the ECT of the preceding activities.

3. $ECT_i = EST_i + D_i$. That is the ECT of any activity is equal to its EST plus its duration.

4. The process of computing the Early Times is called Forward Pass.

The computations are shown in the diagram below.



SESSION 4.9: LATE TIMES

The Late Completion Time (LCT) for node i represented by LCT_i is the latest time at which the event corresponding to node i can occur without delaying the completing of the project. Calculations begin from the ‘finish’ node and move to the ‘start’ node. At each node a number is computed representing the Latest Completion Time (LCT) the corresponding event.

Note:

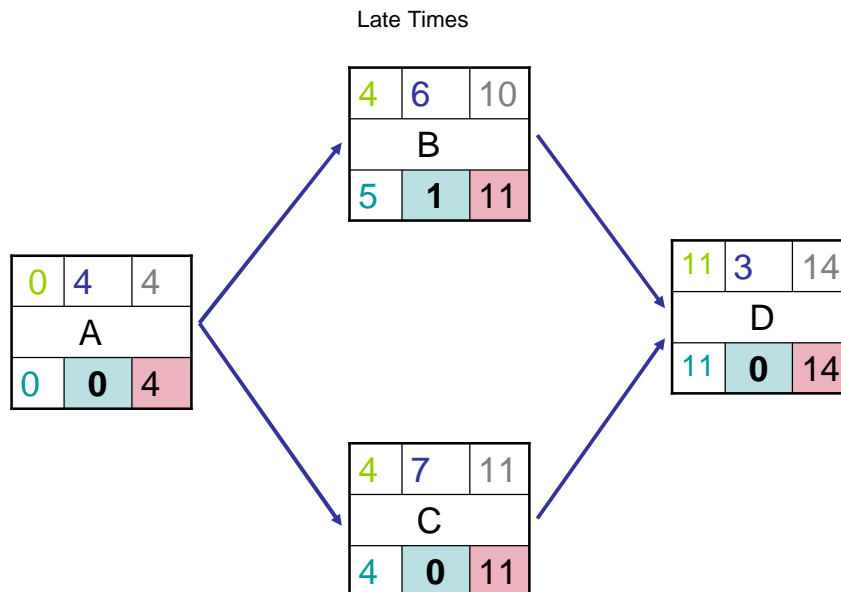
1. $LCT_{FinishNode} = ECT_{FinishNode}$. That is LCT of the last activity in a network diagram is equal to the ECT of the last activity.

2. $LCT_i = \text{Min}(LST_j)$. That is the smallest of the earliest start times for all activities that immediately follow the activity.

3. $LST_i = LCT_i - D_i$. That is the LST of any activity is equal to its LCT minus its duration.

4. The process of computing the Late Times is called Back Pass.

The computations are shown in the diagram below.



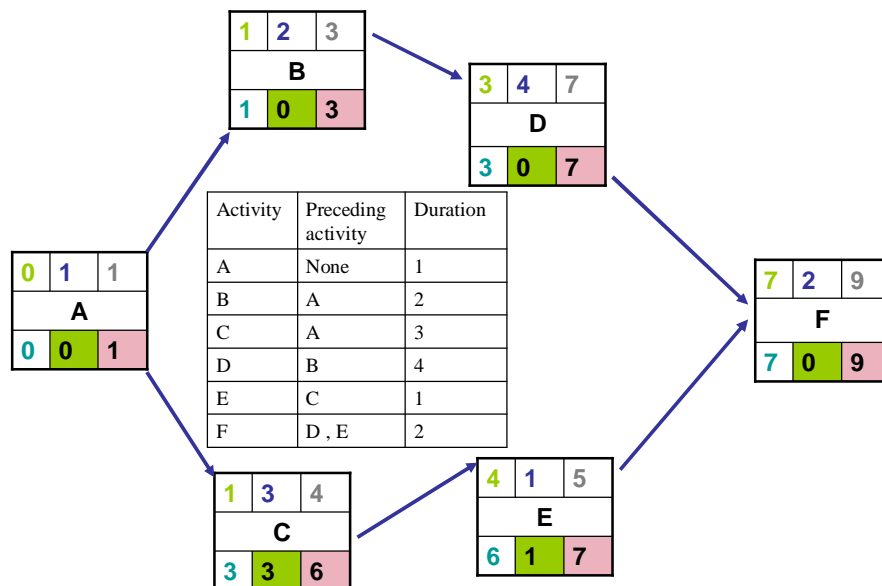
SESSION 4.10: FLOAT

Float (spare time or slack time) is the amount of time a path of activities could be delayed without delaying the overall project. A float can only be associated with activities, which are non-critical. By definition, activities on the critical path cannot have a float (spare time). In other words, the float for an activity is the difference between the maximum time available for that activity and the duration of that activity. The float for an activity with given by:

Float = Latest Start Time – Earliest Start Time (F= LST– EST) or (LCT– ECT)

Example 1

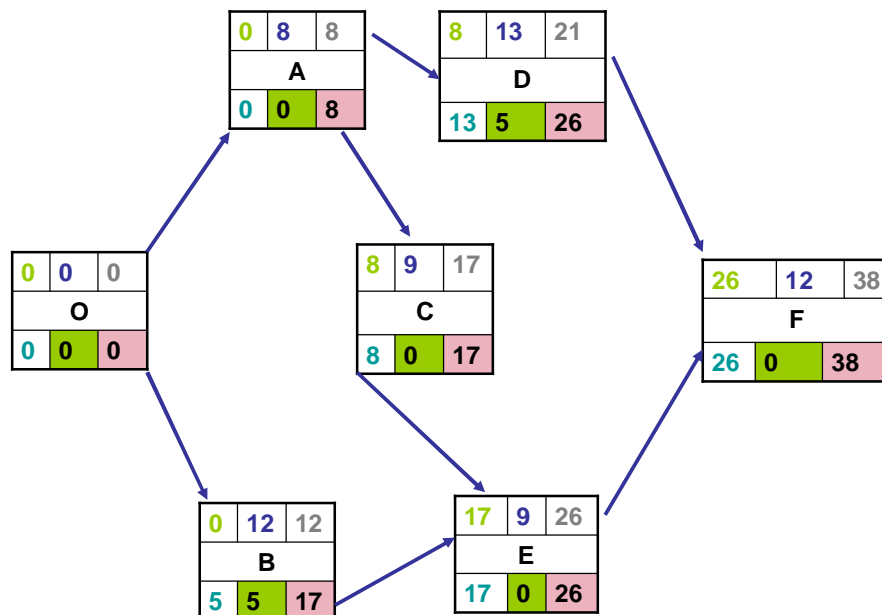
Activity	Preceding activity	Duration
A	None	1
B	A	2
C	A	3
D	B	4
E	C	1
F	D, E	2



EXAMPLE 2

Activity	Preceding activity	Duration
A	-	8
B	-	12
C	A	9
D	A	13
E	B , C	9
F	D , E	12

Solution



SESSION 4.11: PROJECT TIME REDUCTION

One important aspect of project management is looking at ways of reducing the overall project time at an acceptable cost. Reducing or shortening the duration of an activity is known as crashing in the CPM terminology. The normal duration of a project incurs a given cost and by employing more labour, working overtime, more equipment, etc, the duration could be reduced but at the expense of higher costs. Since some ways of reducing the project duration will be cheaper than others, network time/cost analysis seeks to find the cheapest way of reducing the overall duration. A common feature of many projects is a penalty clause for delayed completion and or bonus for earlier completion. Thus, project managers take these into consideration whenever undertaking any project.

We use a network technique known as Critical Path Method (CPM) to enable us to look at the order of activities in a project and to plan the time involved. CPM can be thought of as an application of graph theory. The main objectives of CPM are:

- i. To locate the critical activities.
- ii. To allocate the time of the other activities to obtain the most efficient use of labour and resources.
- iii. To consider ways of reducing the total project time by speeding up the activities on the critical path while monitoring the overall project to ensure that the critical activities remain critical.

There may be more than one critical path in a network and it is possible for the critical path to run through a dummy activity. The critical calculation involves two phases:

- 1. Crashing:** Reducing the duration of a project/activity at a minimum cost. Before we proceed, let us look at the following terminologies.

2. **Normal Cost:** The cost associated with a normal time estimate for an activity. Often, the normal time estimate is set at the point where resources are used in the most efficient manner.
3. **Crash Cost:** The cost associated with the minimum possible time for an activity. Crash costs, because of extra wages, overtime premiums, extra facility costs, are always higher than the normal cost.
4. **Crash Duration/Time:** The minimum time that an activity is planned to take. The minimum time is invariably brought about by the application of extra resources, for example, more labour or machinery.
5. **Cost Slope:** This is the average cost of shortening an activity by one time unit(day, week, month as appropriate). The cost slope is generally assumed to be linear and is calculated as follows:

$$\text{Cost slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}} = \frac{\text{Increase in cost}}{\text{Decrease in time}}$$

Illustration 1: Activity A has the following information:

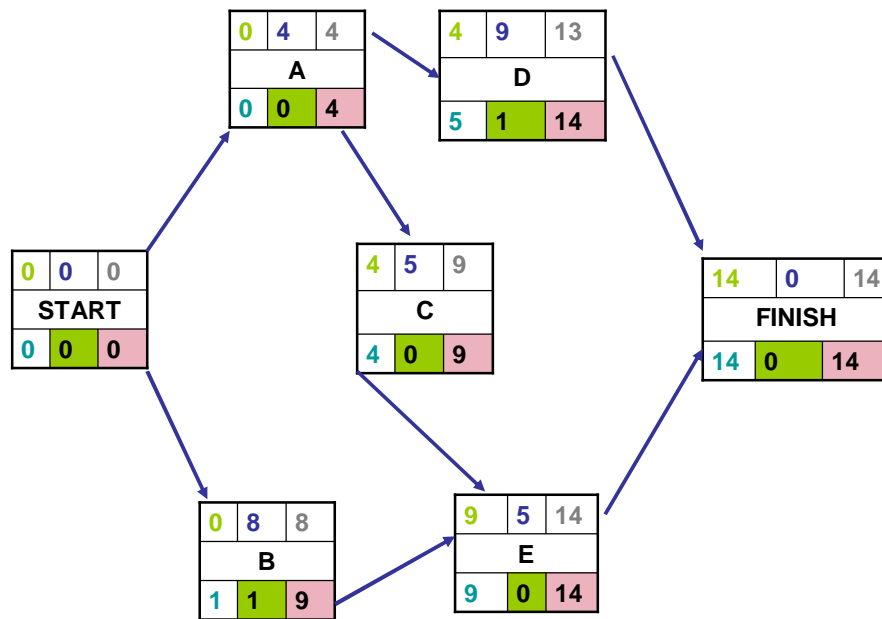
Normal time and cost: 12 days @ \$480

Crash time and cost: 8 days @ \$640

$$\text{Cost slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}} = \frac{\text{Increase in cost}}{\text{Decrease in time}} = \frac{640 - 480}{12 - 8} = \$40/\text{day}$$

Illustration 2: The table below shows the data for a simple project

Activity	Preceding activity	Normal time	Crash time	Normal cost	Crash cost	Cost slope
A	None	4	3	360	460	60
B	None	8	5	300	510	70
C	A	5	3	170	270	50
D	A	9	7	220	300	40
E	B , C	5	3	200	360	80



- Normal project duration = critical path: 1 → 2 → 3 → 4 (i.e. activities A, C, and E) = 4+5+5=14 days.

OR the LCT of the Finish node can be use: LCT = 14 (see network diagram)

- Normal Project Cost = Cost of all activities at normal time
= 360+300+170+220+200 = \$1250

Steps for crashing

1. Draw the network diagram for the project.
2. Determine the critical activities.
3. Calculate the cost slopes for the critical activities.

Note:

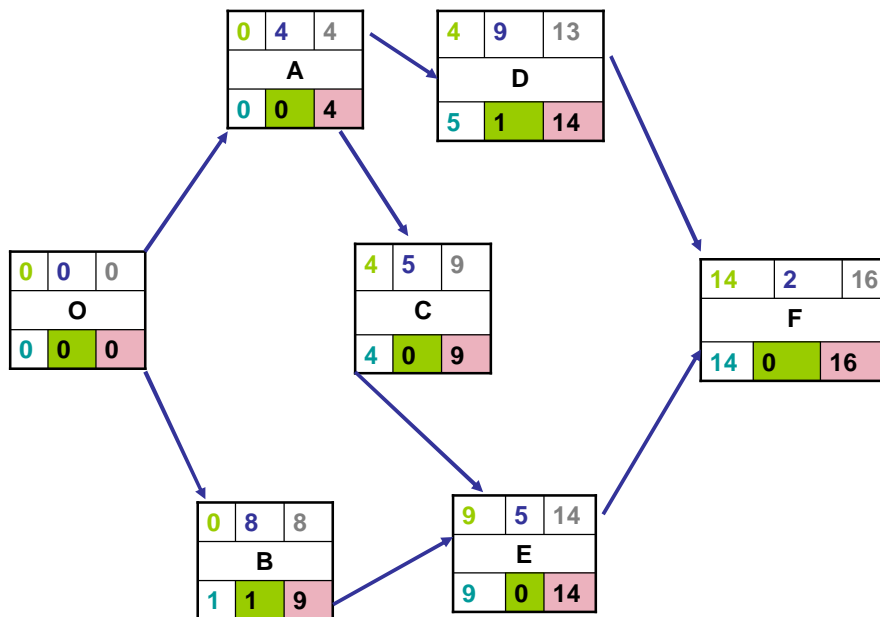
1. **Only** critical activities are crashed in order to reduce the project duration.
2. An activity **cannot** be crashed beyond its crash duration

Illustration.

Crashing:-Example 1

Activity	Pre activity	Normal time	Crash time	Normal cost	Crash cost
A	None	4	3	360	460
B	None	8	5	300	510
C	A	5	3	170	270
D	A	9	7	220	300
E	B, C	5	3	200	360
F	D, E	2	1	200	250

Step 1: Draw the network diagram



Step 2: Determine the critical activities. These are A, C, E and F

Step 3: Calculate the cost slopes for the critical activities

Activity	Pre activity	Normal time	Crash time	Normal cost	Crash cost	Cost slope
A**	None	4	3	360	460	100
B	None	8	5	300	510	
C**	A	5	3	170	270	50
D	A	9	7	220	300	
E**	B , C	5	3	200	360	80
F**	D, E	2	1	200	250	50

Now suppose we want to crash the project by 5 days(that is finish the project in 11 days instead of the 16 days), what will be the cost of the project within this time?

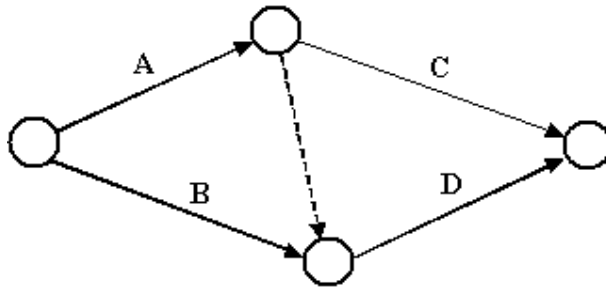
1. The normal project completion time is 16 days(LCT = 16)
2. The cost of completing the project in 16 days is \$1450.00
(360+300+170+220+200+200)
3. Crashing is done as follows;
 - i. Crash activity C by 2 days = $\$50 * 2 = \100.00
 - ii. Crash activity F by 1 day = $\$50 * 1 = \50.00
 - iii. Crash activity E by 2 days = $\$80 * 2 = \160.00
 - iv. Total cost of crashing = $\$310.00(100+50+160)$
 - v. Total cost of completing the project in 11 days
= $\$1780.00$ (Normal cost of project + cost of crashing by 5 days). That is 1450 + 310

SESSION 4.12: ACTIVITY ON ARC (AOA) DIAGRAMS

The table will be used to demonstrate the drawing of AOA diagrams

<u>Activity</u>	<u>Predecessors</u>	<u>Duration</u>
A	None	4 months
B	None	3 months
C	A	2 months
D	A, B	4 months

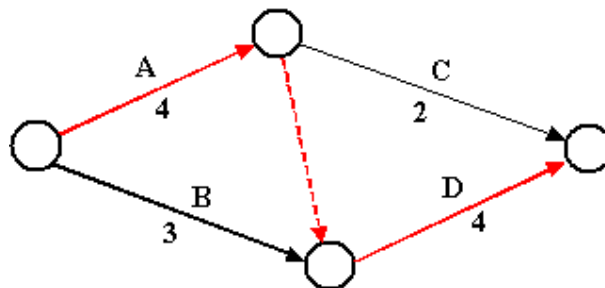
The network diagram for the above table is shown below:



The **dummy activity** (shown by the dotted line) is needed in the AOA network to show the predecessors correctly, i.e. that D depends on both A and B, but C depends only on A. AOA networks often need several dummy activities. AON networks do not need dummy activities.

1. Critical Activities

An activity is said to be critical if its duration cannot be extended without delaying the completion of the project.



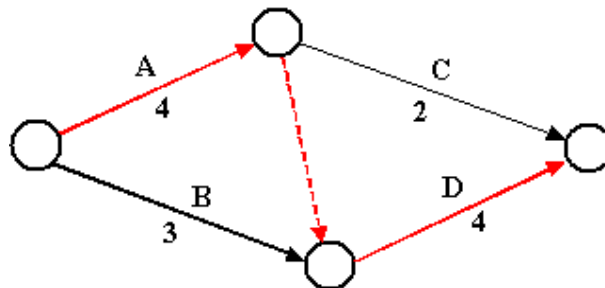
Notice that there are three paths through the network:

Path	A-C	B-D	A-D
Length	6	7	8

The duration of the project is determined by the longest path - D cannot begin until A has been completed. The critical activities are those which lie on a longest path through the network - here A and D are critical. Any extension or delay in a critical activity will make such a path longer still, thereby delaying completion of the project. A longest path through a network is called a critical path. On the diagram the critical path is shown in red.

2. Floats

In the simple network below the critical path is shown in red.

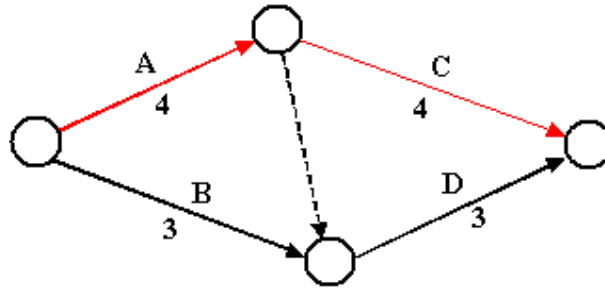


Any activity that is not a critical activity can be extended or delayed by a certain amount without delaying the completion of the project. For example, activity C should start after 4 months and be finished after 6 months. However, the finish could be delayed by 2 months, i.e. until the end of month 8, without affecting the completion time of the project as a whole. We say that activity C has a total float of 2 weeks.

The **Total Float** of an activity is the amount of time by which it may be extended or delayed without delaying completion of the project, assuming no extension or delay in any other activity. A critical activity can be defined as one which has a total float of zero.

There are two other types of float that are sometimes used. To understand these, suppose the durations are changed in the example above. The critical

path is now A-C of duration 8 months, B now has a total float of 2 and D has a total float of 1.



The **Free Float** of an activity takes into consideration the effect on subsequent activities of a delay. If B is delayed by 2 months it removes the spare time that was available for D, since if D was now also delayed by 1 the total project duration would extend to 9 months. However, B can be delayed by 1 month without reducing the spare time for any subsequent activity. Therefore, B has a free float of 1 month. D also has a free float of 1.

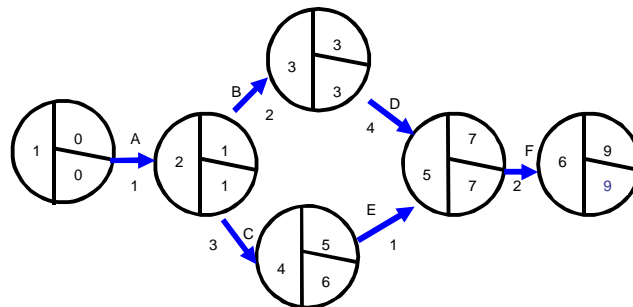
The **Independent Float** of an activity is calculated assuming the worst circumstances, i.e. the activity's predecessors finish at their latest times and we want subsequent activities to begin at their earliest times. If this is possible, and there is still time to spare, then this is called independent float. The free float of 1 month for B is also an independent float. However, assuming the worst scenario in which B is delayed by 2 months then D cannot be delayed at all. Hence D has an independent float of 0.

Example 2.

Draw a network diagram using AOA

Activity	Preceding activity	Duration
A	None	1
B	A	2
C	A	3
D	B	4
E	C	1
F	D , E	2

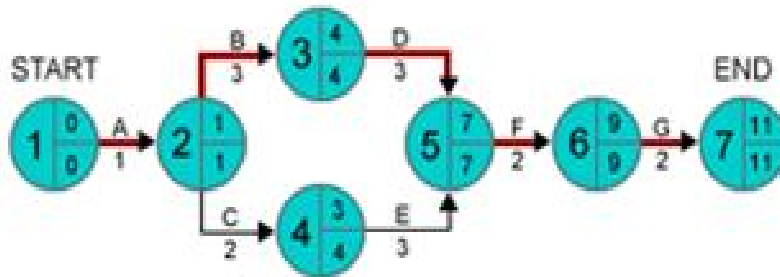
Network diagram-AOA



EXAMPLE 2

Task	Description	Order/Logic	Time
A	Plan Primary Research	To be completed first	1 wks
B	Prepare Mail Shot (Postal Survey)	Start when A is complete	3 wks
C	Prepare Questionnaire	Start when A is complete	2 wks
D	Send and Wait for Mail Shot Replies	Start when B is complete	3 wks
E	Issue Questionnaire	Start when C is complete	3 wks
F	Compile and Analyze Results	Start when D & E is complete	2 wks
G	Plan Selling Campaign	Start when D, E & F is complete	2 wks

EXAMPLE 3



SESSION 4.13: PROGRAM EVALUATION AND REVIEW

TECHNIQUE(PERT)

The **Program Evaluation and Review Technique** commonly abbreviated **PERT** is a model for project management invented by United States Department of Defense's US Navy Special Projects Office in 1958 as part of the Polaris mobile submarine-launched ballistic missile project. This project was a direct response to the Sputnik crisis.

PERT is basically a method for analyzing the tasks involved in completing a given project, especially the time needed to complete each task, and identifying the minimum time needed to complete the total project. It was able to incorporate uncertainty in the sense that it was possible to schedule a project not knowing precisely the details and durations of all the activities. It is more of an event-oriented technique rather than start- and completion-oriented. This technique is used more in R&D-type projects where Cost is not a major factor but Time is.

The Program Evaluation and Review Technique (PERT) is a network model that allows for randomness in activity completion times. It has the potential to reduce both the time and cost required to complete a project.

PERT originally was an *activity on arc* network, in which the activities are represented on the lines and milestones on the nodes. Over time, some people began to use PERT as an *activity on node* network. For this discussion, we will use the original form of activity on node.

A distinguishing feature of PERT is its ability to deal with uncertainty in activity completion times. For each activity, the model usually includes three time estimates:

- *Optimistic time* - generally the shortest time in which the activity can be completed. It is common practice to specify optimistic times to be three standard deviations from the mean so that there is approximately a 1% chance that the activity will be completed within the optimistic time.

- *Most likely time* - the completion time having the highest probability. Note that this time is different from the *expected time*.
- *Pessimistic time* - the longest time that an activity might require. Three standard deviations from the mean is commonly used for the pessimistic time.

PERT assumes a beta probability distribution for the time estimates. For a beta distribution, the expected time for each activity can be approximated using the following weighted average:

$$\text{Expected time} = (\text{Optimistic} + 4 \times \text{Most likely} + \text{Pessimistic}) / 6$$

This expected time may be displayed on the network diagram.

To calculate the variance for each activity completion time, if three standard deviation times were selected for the optimistic and pessimistic times, then there are six standard deviations between them, so the variance is given by:

$$[(\text{Pessimistic} - \text{Optimistic}) / 6]^2$$

4.13.1: Benefits Of PERT

PERT is useful because it provides the following information:

- Expected project completion time.
- Probability of completion before a specified date.
- The critical path activities that directly impact the completion time.
- The activities that have slack time and that can lend resources to critical path activities.
- Activity start and end dates.

4.13.2: Limitations

The following are some of PERT's weaknesses:

- The activity time estimates are somewhat subjective and depend on judgement. In cases where there is little experience in performing an activity, the numbers may be only a guess. In other cases, if the person or group performing the activity estimates the time there may be bias in the estimate.
- Even if the activity times are well-estimated, PERT assumes a beta distribution for these time estimates, but the actual distribution may be different.
- Even if the beta distribution assumption holds, PERT assumes that the probability distribution of the project completion time is the same as the that of the critical path. Because other paths can become the critical path if their associated activities are delayed, PERT consistently underestimates the expected project completion time.

The underestimation of the project completion time due to alternate paths becoming critical is perhaps the most serious of these issues. To overcome this limitation, Monte Carlo simulations can be performed on the network to eliminate this optimistic bias in the expected project completion time.

4.13.3: Terminology

- A *PERT event*: is a point that marks the start or completion of one (or more) tasks. It consumes no time, and uses no resources.
- A PERT event that marks the completion of one (or more) tasks is not “reached” until all of the activities leading to that event have been completed.
- A *predecessor event*: an event (or events) that immediately precedes some other event without any other events intervening. It may be the consequence of more than one activity.

- A *successor event*: an event (or events) that immediately follows some other event without any other events intervening. It may be the consequence of more than one activity.
- A *PERT activity*: is the actual performance of a task. It consumes time, it requires resources (such as labour, materials, space, machinery), and it can be understood as representing the time, effort, and resources required to move from one event to another. A PERT activity cannot be completed until the event preceding it has occurred.
- *Optimistic time* (O): the minimum possible time required to accomplish a task, assuming everything proceeds better than is normally expected
- *Pessimistic time* (P): the maximum possible time required to accomplish a task, assuming everything goes wrong (but excluding major catastrophes).
- *Most likely time* (M): the best estimate of the time required to accomplish a task, assuming everything proceeds as normal.
- *Expected time* (T_E): the best estimate of the time required to accomplish a task, assuming everything proceeds as normal (the implication being that the expected time is the average time the task would require if the task were repeated on a number of occasions over an extended period of time).

$$T_E = (O + 4M + P) \div 6$$

- *Critical Path*: the longest pathway taken from the initial event to the terminal event. It determines the total calendar time required for the project; and, therefore, any time delays along the critical path will delay the reaching of the terminal event by at least the same amount.
- *Lead time*: the time by which a *predecessor event* must be completed in order to allow sufficient time for the activities that must elapse before a specific PERT event is reached to be completed.

- *Lag time*: the earliest time by which a *successor event* can follow a specific PERT event.
- *Slack*: the slack of an event is a measure of the excess time and resources available in achieving this event. Positive slack would indicate *ahead of schedule*; negative slack would indicate *behind schedule*; and zero slack would indicate *on schedule*

4.13.4: Steps In The PERT Planning Process

PERT planning involves the following steps:

1. Identify the specific activities and milestones.
2. Determine the proper sequence of the activities.
3. Construct a network diagram.
4. Estimate the time required for each activity.
5. Determine the *critical path*.
6. Update the PERT chart as the project progresses.

Illustrative Example

The following represents a project that should be scheduled using PERT

Activity	Predecessors	Time estimates		
		a	m	b
A	-	10	22	28
B	A	4	4	10
C	A	4	6	14
D	B	1	2	3
E	C,D	1	5	9
F	C,D	7	8	9
G	E,F	2	2	2

1. Calculate the expected time for each activity. This is given as follows: $E(t) = \frac{a + 4m + b}{6}$. Use the expected times to draw the network diagram
2. Determine the critical path. Using the expected times, a critical path is calculated in the same manner as in the single time estimate
3. Calculate the variance(σ^2) for the activity time: Specifically, this is the variance associated with each E(t) and is computed using the formula below: $\sigma^2 = \left(\frac{b-a}{6}\right)^2$
4. Determine the probability of completing the project on a given date. A valuable feature of using three time estimates is that it enables the analyst to assess the effect of uncertainty on project completion time. The mechanics of deriving the probability are as follows:
 - i. Sum the variance values associated with each activity.

- ii. Substitute this figure along with the project due date and the project expected completion time into the transformation formula. This

formula is:
$$z = \frac{D - E(t)}{\sqrt{\sum \sigma^2_{cc}}}$$

Where:

D = Desired completion date for the project

E(t) = Expected completion date for the project

$\sum \sigma^2_{cc}$ = Sum of variances along the critical path.

- iii. Calculate the value of Z, which is the number of standard deviations the project date is from the expected completion time.
- iv. Using the value of Z, find the probability of meeting the project due date (using a table of normal probabilities)

Activity	Predecessors	Time estimates			Expected Time $E(t) = \frac{a + 4m + b}{6}$	Activity Variance $\sigma^2 = \left(\frac{b - a}{6}\right)^2$
		a	m	b		
A	-	10	22	28	21	9
B	A	4	4	10	5	1
C	A	4	6	14	7	$2\frac{2}{9}$
D	B	1	2	3	2	$\frac{1}{9}$
E	C,D	1	5	9	5	$1\frac{7}{9}$
F	C,D	7	8	9	8	$\frac{1}{9}$
G	E,F	2	2	2	2	0

Since there are two critical paths in the network(the above example), we must decide which variances to use in arriving at the probability of meeting the project due date. A conservative approach dictates using the path with the largest variance since this would focus management's attention on the activities most likely to exhibit broad variations. On this basis, the variances associated activities A, C, F, and G would be used to find the probability of completion. Thus:

$$\sum \sigma^2 = 9 + 2\frac{2}{9} + \frac{1}{9} + 0 = 11.89$$

Suppose management asks for the probability of completing the project in 35 weeks, then $D = 35$. The expected completion time was found to be 38. Substituting these values in Z, we have:

$$z = \frac{D - E(t)}{\sqrt{\sum \sigma^2_{cc}}} = \frac{35 - 38}{\sqrt{11.89}} = -0.87$$

Using a table of normal probabilities, we see that a Z value of -0.87 yields a probability of 0.19, which means that the project manager has only 19% chance of completing the project in 35 weeks.

SELF ASSESSMENT QUESTIONS

QUESTION 1

The table below shows data for a simple project.

ACTIVITY	PRECEDING ACTIVITY	DURATION	COST	CRASH DURATION	COST
A	-	5	100	4	200
B	-	4	120	2	160
C	A	10	400	4	1000
D	B	7	300	3	700
E	B	11	200	10	250
F	C	8	400	6	800
G	D, E, F	4	300	4	300

- i. Draw a network diagram for the project above.
- ii. Find the critical path.
- iii. Find the normal duration time of the project.
- iv. Find the cost associated with the normal duration time.
- v. Find the cost associated with crashing for three. days

QUESTION 2

The table below shows data for a simple project.

Activity	Preceding activity	Normal duration	Normal cost	Crash duration	Crash cost
A	-	3	1200	2	1500
B	-	2	1250	1	1300
C	A	6	1000	4	1800
D	A	5	1500	3	2800
E	C, D	5	1800	4	3000
F	B, D	3	500	1	1000
G	F	2	500	1	1000

- i. Draw a Network diagram for the project using (a) Activity on arc notation (AOA) and (b) activity on node notation (AON)
- ii. Find the critical path.
- i. Find the normal duration time of the project.
- ii. Find the cost associated with the normal duration time.
- iii. Find the shortest possible completion time of the project.
- vi. Find the cost associated with the shortest possible completion time

QUESTION 3

The table below gives data for a simple project

Activity	Preceding activity	Normal duration	Normal cost	Crash duration	Crash cost
A	-	4	25	3	37
B	A	4	16	4	16
C	-	5	35	2	75
D	A , C	8	49	7	95
E	B , C	8	60	4	130
F	D ,E	6	41	3	76
G	E ,F	5	23	3	45
H	G	4	40	3	65

- i. Calculate and state the critical path(s) and the cost of completion in the normal time
- ii. Calculate and state the critical path(s) and the cost of completion in the crash time
- iii. Calculate and state the minimum cost of completion and associated critical path(s).

QUESTION 4

The table below gives data for a simple project

Activity	Preceding activity	Normal duration	Normal cost	Crash duration	Crash cost
A	-	8	100	6	200
B	-	4	150	2	350
C	A	2	50	1	90
D	A	10	100	5	400
E	B	5	100	1	200
F	C , E	3	80	1	100

- i. Draw a Network diagram for the project using activity on node notation (AON)
- ii. Find the critical path.

- iii. Find the normal duration time of the project.
- iv. Find the cost associated with the normal duration time.
- v. Find the shortest possible completion time of the project.
- vi. Find the cost associated with the shortest possible completion.

QUESTION 5

The Ressembler Group is looking at the possible test launch of a new type of picture frame called 'Dale'. The main activities have been identified. Times estimated in weeks and costs estimated in dollars are shown below.

.Activity	Preceding Activity	NORMAL		CRASH	
		Time (weeks)	Cost (\$)	Time (weeks)	Cost (\$)
A	-	4	10,000	2	11,000
B	A	3	6,000	2	9,000
C	A	2	4,000	1	6,000
D	B	5	14,000	3	18,000
E	B, C	1	9,000	1	9,000
F	C	3	7,000	2	8,000
G	E, F	4	13,000	2	25,000
H	D, E	4	11,000	1	18,000
I	H, G	6	20,000	5	29,000

(a) Draw the network for this project. (Use AOA notation)

- i. State the critical path and critical activities
- ii. Find the normal project time.
- iii. Find the normal project cost.

(b). The project director decides to shorten the project duration by three (3) weeks.

- i. What activities should be crashed?
- ii. What will be the total cost of the project in this time?

QUESTION 6

A manufacturing concern has received a special order for a number of units of a special product that consists of two components parts: X and Y. The product is a nonstandard item that the firm has never produced before, and scheduling personnel have decided that the application of CPM is warranted. A team of manufacturing engineers has prepared the following table.

Activity	Description	Predecessors	Expected Time(days)
A	Plan production	-	5
B	Procure materials for part X	A	14
C	Manufacture part X	B	9
D	Procure materials for part Y	A	15
E	Manufacture part Y	D	10
F	Assemble parts X and Y	C, E	4
G	Inspect assemblies	F	2
H	Completed	G	0

- Construct a graphic representation of the data above
- Identify the critical path
- What is the length of time to complete the project?
- Find the floats
- Find the possible number of days by which the project duration can be reduced and state the activities that need to be reduced.

QUESTION 7

The following represents a project that should be scheduled using PERT

Activity	Immediate predecessor(s)	Time(Days)		
		a	m	b
A	-	1	3	5
B	A	1	2	3
C	A	1	2	7
D	A	2	3	4
E	B	3	4	11
F	C , D	3	4	5
G	D , E	1	4	6
H	F , G	2	4	5

- Draw the network diagram
- What is the critical path?
- What is the expected project completion time?
- What is the probability of completing the project within 16 days?

QUESTION 8

The table below represents a plan for a project

Job No.	Immediate predecessor(s)	Time(Days)		
		a	m	b
1	-	2	3	4
2	1	1	2	3
3	1	4	5	12
4	1	3	4	11
5	2	1	3	5
6	3	1	2	5
7	4	1	8	9
8	5 , 6	2	4	6
9	8	2	4	2
10	7	3	4	5
11	9 ,10	5	7	8

- construct an appropriate network diagram
- Indicate the critical path
- What is the expected completion time for the project?